Weekly Problem Sets #5

Academic Economics; [Discord Link]

Due by August 28th, 2021

1 Graduate Problem

Suppose that a bank has lent L to a firm that has invested in a risky technology with a net (unit) return $\tilde{y}$. In the absence of collateral, the net (unit) return to the bank will be $\min(r_L, \tilde{y})$. When $\tilde{y} < r_L$, the firm defaults, and the bank seizes the firm's assets, which are worth $(1 + \tilde{y})L$. For all parts the profit of the bank is,

$$\tilde{\pi}(L, D, \tilde{y}) = [min(r_L(L), \tilde{y}) - r]L + [r - r_D(D)]D$$

1. Assuming the bank has no equity, show that the bank itself will default if $\tilde{y}$ is below some threshold y^*. Compute this value.
2. Assume risk neutrality and limited liability of the bank. The bank chooses the volumes L^* of loans and D^* of deposits that maximize the expectation of the positive part of its profit. (If the profit is negative, the bank defaults.) Write the first order conditions that characterize L^* and D^*.
3. Show that L^* depends in general on what happens on the deposit side.

1.1 Question 1 Answer

From Freixas and Rochet, Microeconomics of Banking.

1. The threshold y^* corresponds to the value of $\tilde{y}$ such that $\tilde{\pi} \leq 0$ for a given L and D. this implies

$$0 = (y^* - r)L + [r - r_D(D)]D$$

Or

$$y^* = r - [r - r_D(D)]\frac{D}{L}$$

2. The objection function of the bank becomes
$\pi(L, D) = E[\max(0, \tilde{\pi}(L, D, \tilde{y})]$. The first order conditions are

$$\frac{\partial \pi}{\partial D} = E[\frac{\partial \tilde{\pi}}{\partial D}(L, D, \tilde{y})I\{\tilde{y} > y^*] = 0$$
$$\frac{\partial \pi}{\partial L} = E[\frac{\partial \tilde{\pi}}{\partial L}(L, D, \tilde{y})I\{\tilde{y} > y^*] = 0$$

where

$$\frac{\partial \tilde{\pi}}{\partial D}(L, D, \tilde{y}) = r - r_D - Dr_D'(D)$$

and

$$\frac{\partial \tilde{\pi}}{\partial L}(L, D, \tilde{y}) = \begin{cases} r_L - r + Lr'(L) & \tilde{y} > r_L \\ \tilde{y} - r & \tilde{y} < r_L \end{cases}$$

3. The first order condition becomes,

$$0 = \frac{\partial \pi}{\partial L} = (r_L - r + Lr_L')P(\tilde{y} > r_L) + E[(\tilde{y} - r)I\{y^* < \tilde{y} < r_L\}]$$

The second term, if it does not vanish, introduces a relationship between deposites and loans, since y^* depends on D per part 1.

2 Undergraduate Problem

From Pindyck and Rubinfeld, "Microeconomics, 7th ed."

As a chairman of the board of ASP industries, you estimate that your annual profit is given by the table below. Profit (P) is conditional upon market demand and the effort of your new CEO. The probabilities of each demand condition occuring are also shown in the table.

Market demand	Low Demand	Medium Demand	High demand
Market probabilities	0.30	0.40	0.30
Low effort	$\pi =$ \$5 million	$\pi =$ \$10 million	$\pi =$ \$15 million
High effort	$\pi =$ \$10 million	$\pi =$ \$15 million	$\pi =$ \$17 million

You must design a compensation package for the CEO taht will maximize the firm's expected profit. While the firm is risk neutral, the CEO is risk averse. The CEO's utility function is

$$U = W^{.5} - 100 * (1 - 1\{\text{High Effort}\})$$

Where W is the CEO's income. You know the CEO's utility function, and both you and the CEO know all the information in the preceeding table. You do not know the level of the CEO's effort at time of compensation or the exact state of demand. You do see the firm's profit, however.

Of the three alternative compensation packages below, which do you prefer? **WHY**?

1. Pay the CEO a flat salary of \$575,00 per year
2. Pay the CEO a fixed 6 percent of yearly firm profits
3. Pay the CEO a flat salary of \$500,000 per year and then 50 percent of any firm profits above \$15 million.

2.1 Question 2 Answer

The firm's profit is assumed to be

$$\pi - W$$

and the two incentive compatibility constraints to provide high effort require that

$$E[W_H^{.5}] - 100 \geq E[W_L]$$

$$E[\pi^H - W^H] \geq E[\pi^L - W^L]$$

The first states that the CEO earns more providing high effort than low effort, and the second is that the bank's profit is higher when high effort is provided at a given wage rate.

1. For any flat salary there is no incentive for a CEO to increase their effort level, since $W_H = W_L$, and so providing effort only decreases CEO utility.
2. At 6 percent of profits, the expected salary for low effort is,

$$E[w^L] = E[\pi^L] * 0.06 = (5 * .3 + 10 * .4 + 15 * .3) * 0.06 = 10 * .06 = \$600,000$$

and for high effort is

$$E[w^H] = E[\pi^H] * 0.06 = (10 * .3 + 15 * .4 + 17 * .3) * 0.06 = 14.1 * 0.06 = \$846,000$$

At each wage level one can check that the incentive compatibility constraint is met, so the CEO is induced to provide high effort. The value of the extra salary to the firm is less than the benefit of high demand, so they're willing to offer the salary. The dominant strategy for the CEO is also to always provide high effort.

3. The CEO wants to provide low effort when there is both low and medium demand. The extra salary they generate during high demand is \$1,000,000. The CEO's incentive compatibility constraint binds here, as

$$E[U|\text{high effort}] = (500000)^{.5} * (0.7) + (1500000)^{.5} * (0.3) - 100 = 762.398$$

$$E[U|\text{low effort}] = (500000)^{.5} = 707.106781187$$

And the firm's incentive compatibility constraint is,

$$(10 * .3 + 15 * .4 + 17 * .3) - .5 * .7 - 1.5 * .3 > (5 * .3 + 10 * .4 + 15 * .3) - .5$$
$$13.3 > 9.5$$

As calculated, the expected salary in the first case is \$846,000, and in the second is \$800,000. The third option is the lowest cost that meets both the firm and CEO's incentive compatibility constraints.