Weekly Problem Set: August 24th, 2021

Submit answers by direct messaging @ModMail on the Academic Economics Discord
or emailing academicecondiscord@gmail.com
Solve a problem for "Extra Credit" or "Novice Extra Credit" server tag for two weeks

Due before August 28th, 2021 @ 12:00 pm EST when answers are posted

1 Extra Credit Problem

1. Let X, Y, Z be real-valued random variables. Assume further that $X, Z \in \{0, 1\}$, and $E[Y^2] < \infty$. Is it true that the Best Linear Predictor (BLP) of Y on X, Z is equal to $E[Y \mid X, Z]$? For clarity, note that a constant is included in the best linear predictor as well. Explain briefly.

2. Let $Y = \alpha + X\beta + u$, but $E[Xu] \neq 0$. Let there be an instrument Z, but it's imperfect, such that the linear projection $E[Zu] \neq 0$.

   - Characterize $\hat{\beta}_{OLS}$ and $\hat{\beta}_{IV}$ (this should be a single scalar)
   - Assume that y_i, x_i, z_i are stationary and weakly dependent, and that both the endogenous regressor and the imperfect IV are correlated with Y in the same direction. Show if $\sigma_{xz} < 0$, then what is the relationship between β, β_{IV}, and β_{OLS}?

2 Novice Extra Credit Problem

Let Y_1, Y_2, Y_3, Y_4 be independent, identically distributed random variables from a population with mean μ and variance σ^2. Let $\bar{Y} = \frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4)$ denote the average of these four random variables.

1. What are the expected value and variance of $\bar{Y}$ in terms of μ and σ^2?

2. Now, consider a different estimator of μ,

$$W = \frac{1}{8}Y_1 + \frac{1}{8}Y_2 + \frac{1}{4}Y_3 + \frac{1}{2}Y_4$$

   This is an example of a weighted average of the Y_i. Show that W is also an unbiased estimator of μ. Find the variance of W.

3. Based on your answers to parts 1 and 2, which estimator of μ do you prefer, $\bar{Y}$ or W?